Geog 480: Principles of GIS

Fundamental spatial concepts - II

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**Topology of space**

- **Topology**: “study of form”; concerns properties that are invariant under topological transformations
- **Intuitively**, topological transformations are rubber sheet transformations

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<th>Topological</th>
<th>Non-topological</th>
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<td>A point is at an end-point of an arc</td>
<td>Distance between two points</td>
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<td>A point is on the boundary of an area</td>
<td>Bearing of one point from another point</td>
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<td>A point is in the interior/exterior of an area</td>
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<td>An arc is simple</td>
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<td>An area is open/closed/simple</td>
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Point set topology

• One way of defining a topological space is with the idea of a *neighborhood*

• Let $S$ be a given set of points. A *topological space* is a collection of subsets of $S$, called *neighborhoods*, that satisfy the following two conditions:
  ○ $T1$ Every point in $S$ is in some neighborhood.
  ○ $T2$ The intersection of any two neighborhoods of any point $x$ in $S$ contains a neighborhood of $x$

• Points in the Cartesian plane and *open disks* (circles surrounding the points) form a topology
Point set topology

Each point is in a neighborhood.

The intersection of two neighborhoods of a point contains a neighborhood of that point.
Usual topology

- **Usual topology**: naturally comes to mind with Euclidean plane and corresponds to the rubber-sheet topology
  - Validate $T_1$ and $T_2$
Travel time topology

- Let $S$ be the set of points in a region of the plane
- Suppose:
  - the region contains a transportation network and
  - we know the average travel time between any two points in the region using the network, following the optimal route
- Assume travel time relation is symmetric
- For each time $t$ greater than zero, define a $t$-zone around point $x$ to be the set of all points reachable from $x$ in less than time $t$
Travel time topology

- Let the neighborhoods be all $t$-zones around a point
- Verify T1 and T2
Near point

- Let $S$ be a topological space
- Then $S$ has a set of neighborhoods associated with it. Let $C$ be a subset of points in $S$ and $c$ an individual point in $S$
- Define $c$ to be near $C$ if every neighborhood of $c$ contains some point of $C$
Properties of a topological space

- **Open set**
  - Every point of a set can be surrounded by a neighborhood that is entirely within the set

- **Closed set**
  - A set contains all its near points

- **Closure ($X^-$)**
  - The union of a point set with the set of all its near points
Properties of a topological space

• **Interior** ($X^o$) of a point set
  - Consists of all points that belong to the set and are not near points of the complement of the set

• **Boundary of a point set** ($\partial X$)
  - Consists of all points that are near to both the set and its complement

• **Connectedness**
  - Partition into two non-empty disjoint subsets: A and B
  - Either A contains a point near B
  - Or B contains a point near A
Open and closed sets

- Let $S$ be a topological space and $X$ be a subset of points of $S$.
  - Then $X$ is open if every point of $X$ can be surrounded by a neighborhood that is entirely within $X$
    - A set that does not contain its boundary
  - Then $X$ is closed if it contains all its near points
    - A set that does contain its boundary
Closure, boundary, interior

- Let $S$ be a topological space and $X$ be a subset of points of $S$

  - The **closure** of $X$ is the union of $X$ with the set of all its near points
    - denoted $X^-$
  - The **interior** of $X$ consists of all points which belong to $X$ and are not near points of $X'$
    - denoted $X^°$
  - The **boundary** of $X$ consists of all points which are near to both $X$ and $X'$. The boundary of set $X$ is denoted $\partial X$
Topology and embedding space

2-space

1-space
Topological invariants

- Properties that are preserved by topological transformations are called *topological invariants*
Connectedness

- Let $S$ be a topological space and $X$ be a subset of points of $S$
- Then $X$ is **connected** if whenever it is partitioned into two non-empty disjoint subsets, $A$ and $B$,
  - either $A$ contains a point near $B$, or $B$ contains a point near $A$, or both
- A set in a topological space is **path-connected** if any two points in the set can be joined by a path that lies wholly in the set
Connectedness

- A set $X$ in the Euclidean plane with the usual topology is *weakly connected* if it is possible to transform $X$ into an unconnected set by the removal of a finite number of points.
- A set $X$ in the Euclidean plane with the usual topology is *strongly connected* if it is connected and not weakly connected.
Connectedness

disconnected

Strongly connected  Weakly connected
Network spaces - abstract graphs

- A **graph** $G$ is defined as a finite non-empty set of **nodes** together with a set of unordered pairs of distinct nodes (called **edges**)
  - Highly abstract
  - Represents connectedness between elements of the space

- Directed graph

- Labeled graph
Abstract graphs

- Connected graph
- Edges
- Path
- Cycle
- Nodes
- Degree
- Isomorphic
- Directed/ non-directed
Tree

- Connected graph
- Acyclic
- Non-isomorphic
Rooted tree

- Root
- Immediate descendants
- Leaf
Metric space - definition

- A point-set $S$ is a *metric space* if there is a distance function $d$, which takes ordered pairs $(s,t)$ of elements of $S$ and returns a distance that satisfies the following conditions:
  - For each pair $s, t$ in $S$, $d(s,t) > 0$ if $s$ and $t$ are distinct points and $d(s,t) = 0$ if $s$ and $t$ are identical.
  - For each pair $s,t$ in $S$, the distance from $s$ to $t$ is equal to the distance from $t$ to $s$, $d(s,t) = d(t,s)$.
  - For each triple $s,t,u$ in $S$, the sum of the distances from $s$ to $t$ and from $t$ to $u$ is always at least as large as the distance from $s$ to $u$. 
Distances defined on the globe

- Metric space
  - Geodesic distance
  - Manhattan distance
  - Travel time distance
- Quasimetric
  - Lexicographic distance
• End of this topic